Lettres grecques et symboles mathématiques

α alpha β beta γ gamma δ delta ε epsilon ζ zeta

η eta

θ theta ι iota κ kappa λ lambda μ mu ν nu

ξ xi

o omicron π pi ρ rho σ sigma τ tau

υ upsilon

φ phi χ chi ψ psi ω omega Γ Gamma Δ Delta

Θ Theta Λ Lambda Ξ Xi Π Pi

Σ Sigma Υ Upsilon Φ Phi Ψ Psi Ω Omega

$\forall$ Pour tout $\exists$ Il existe $\Rightarrow$ Implique $\Leftrightarrow$ Equivalent

$\cap$ Intersection $\cup$ Réunion ϕ vide $\in$ appartient

$\subset$ est inclus

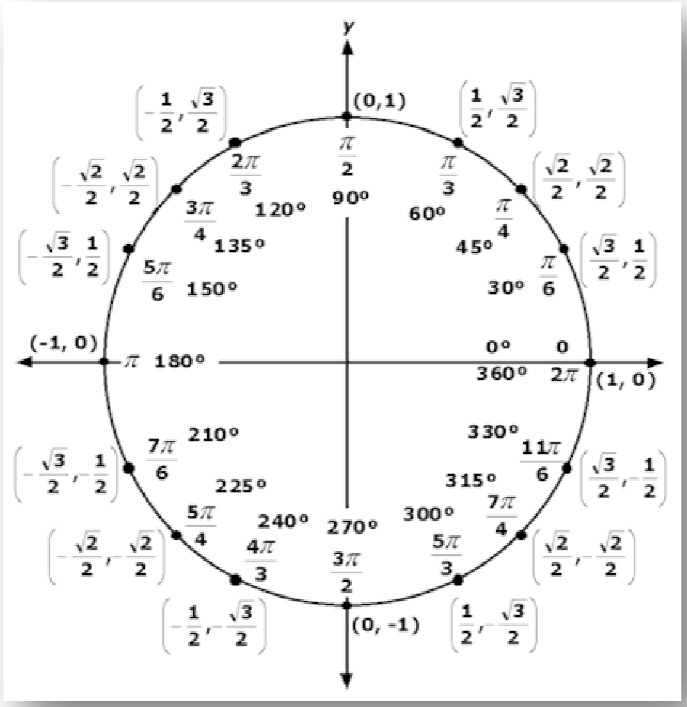
تلخيص في مادة الرياضيات

الدوال المثلثية ودوالها العكسية

الدائرة المثلثية

الفاصلة هي $\cos(\theta)$ و الترتيبة هي $\sin(\theta)$

$A(\cos(\theta),\sin(\theta))$



الدالة Y= Sin(x)

النشر المحدود للدالة

$$\sin(x) = \sum_{n\geq 0} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \cdots + \frac{(-1)^p x^{2p+1}}{(2p+1)!} + o(x^{2p+2}),\ -\infty < x < \infty$$

الشكل الأسي

$$\sin(x) = \frac{e^{ix} - e^{-ix}}{2i}$$

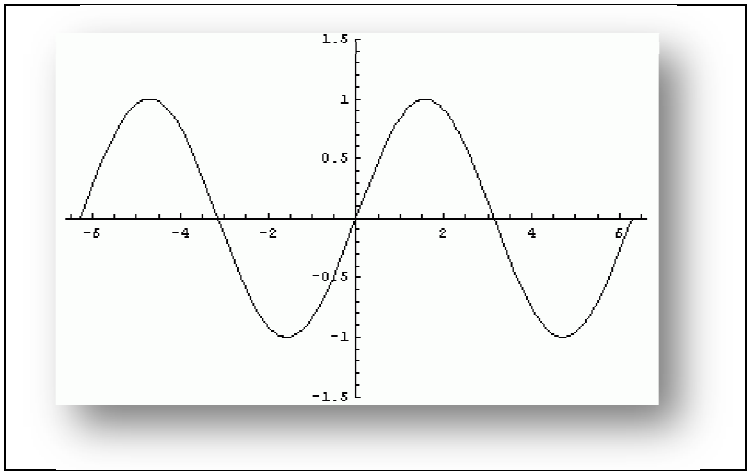
المشتق

$$\frac{dy}{dx} = \cos(x)$$

الرسم البياني

Sin(x) :[-π/2;π/2]➔[-1 ;1]

X ➔ sin(x)



بعض العلاقات التي تخص الدالة sin(x)

$\sin^2(\alpha) = 1 - \cos^2(\alpha)$

$= \frac{1}{2}(1 - \cos(2\alpha))$

Sin (0) =0

$\sin(-\theta)$ $= -\sin(\theta)$ ... ... (fonction impair)

$\text{Sin}(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \sin(\beta)\cos(\alpha)$

$Sin\ (2\alpha) = 2sin(\alpha)cos(\alpha)$

$= \frac{2tan\ (\alpha)}{1+tan^2(\alpha)}$

Sin (3α) = 3Sin (α)-$4\sin^3(\alpha)$

$\text{Sin}(\pi \mp \theta) = \pm\sin(\theta)$

$\text{Sin}(\alpha)+\sin(\beta) = 2\sin(\frac{\alpha+\beta}{2})cos(\frac{\alpha-\beta}{2})$

$\text{Sin}(\alpha)-\sin(\beta) = 2\cos(\frac{\alpha+\beta}{2})sin(\frac{\alpha-\beta}{2})$

$\text{Sin}(\alpha)\sin(\beta) = \frac{1}{2}[\cos(\alpha-\beta) - cos(\alpha+\beta)]$

دالتها العكسية هي Y=$\sin^{-1}(x)$=Arcsin(x)

النشر المحدود للدالة

$\text{Arcsin(x)} = x + \frac{1.x^3}{2.3} + \frac{1.3.x^5}{2.4.5} + \frac{1.3.5x^7}{2.4.6.7} + \cdots + \frac{1.3.5.....(2p-1)x^{2p+1}}{2.4.6.....2p\ \ (2p+1)} + o(x^{2p+2}),$

$-\infty < x < \infty$

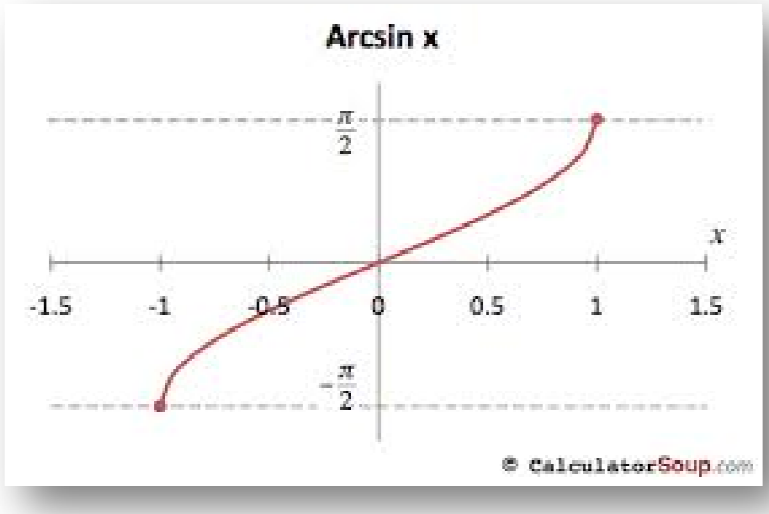
المشتق

$$\frac{dy}{dx} = \frac{1}{\sqrt[2]{1-x^2}}$$

الرسم البياني

Arcsin(x): [-1 ;1] ➔ *[-π/2;π/2]*

X ➔ arcsin (x)



Y=Cos(x)الدالة

النشر المحدود للدالة

$$\cos(x) = \sum_{n\geq 0} \frac{(-1)^n x^{2n}}{2n!} = 1 - \frac{x^2}{2!} + \cdots + \frac{(-1)^p x^{2p}}{2p!} + o(x^{2p+1}),\ -\infty < x < \infty$$

الشكل الأسي

$$\cos(x) = \frac{e^{ix} + e^{-ix}}{2}$$

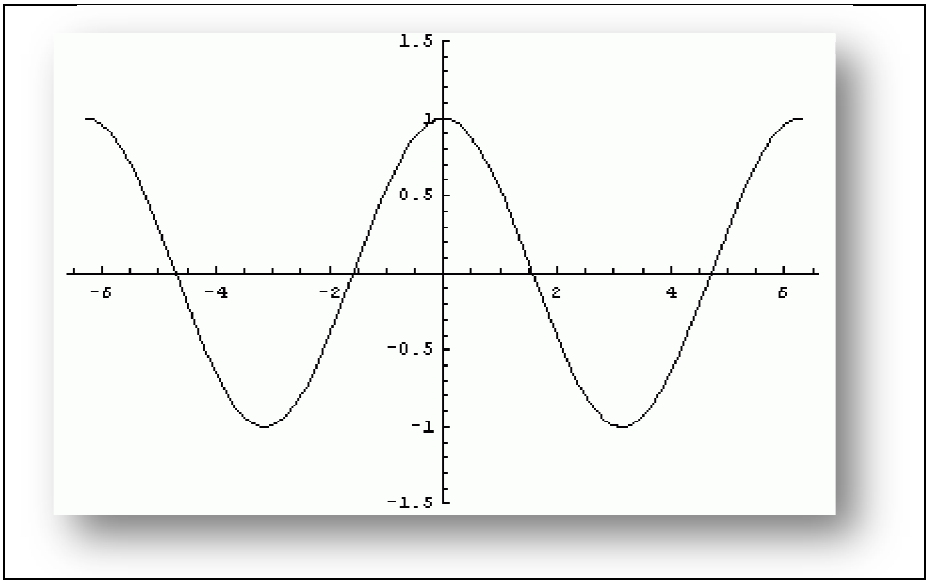
المشتق

$$\frac{dy}{dx} = -\sin(x)$$

الرسم البياني

Cos(x):[0; π] ➔ *[-1;1]*

X ➔ cos (x)



بعض العلاقات التي تخص الدالة Cos(x)

$cos^2(\alpha) = 1 - sin^2(\alpha)$

$= \frac{1}{2}(1 + cos(2\alpha))$

$\cos(0) = 1$

$\cos(-\theta) = \cos(\theta)$ ... ... (fonction pair)

$cos(\alpha \pm \beta) = cos(\alpha)cos(\beta) \mp sin(\beta)cos(\alpha)$

$cos(2\alpha) = cos^2(\alpha) - sin^2(\alpha)$

$= 2cos^2(\alpha) - 1$

$= 1 - 2\,sin^2(\alpha)$

$= \frac{1 - tan^2(\alpha)}{1 + tan^2(\alpha)}$

$\cos(3\alpha) = -3\cos(\alpha) + 4\cos^3(\alpha)$

$cos(\pi \mp \theta) = -cos(\theta)$

$cos(\alpha) + cos(\beta) = 2cos(\frac{\alpha+\beta}{2})cos(\frac{\alpha-\beta}{2})$

$cos(\alpha) - cos(\beta) = -2sin(\frac{\alpha+\beta}{2})sin(\frac{\alpha-\beta}{2})$

$cos(\alpha)cos(\beta) = \frac{1}{2}[cos(\alpha - \beta) + cos(\alpha + \beta)]$

بعض العلاقات التي تربطهما sin و cos

$\sin^2(\alpha) + \cos^2(\alpha) = 1$

$\sin(\alpha)\cos(\beta) = \frac{1}{2}[\sin(\alpha + \beta) + \sin(\alpha - \beta)]$

$\cos(\alpha)\sin(\beta) = \frac{1}{2}[\sin(\alpha + \beta) - \sin(\alpha - \beta)]$

$cos(\frac{\pi}{2} \pm \theta) = \mp sin(\theta)$

$\sin(\frac{\pi}{2} \pm \theta) = \cos(\theta)$

$Y=\cos^{-1}(x)$=Arccos(x) دالتها العكسية هي

النشر المحدود للدالة

$$\text{Arccos}(x)= \frac{\pi}{2} - x - \frac{1.x^3}{2.3} - \frac{1.3.x^5}{2.4.5} + \frac{1.3.5x^7}{2.4.6.7} - \cdots - \frac{1.3.5.....(2p-1)x^{2p+1}}{2.4.6.....2p \quad (2p+1)} + o(x^{2p+2}),$$
$$-\infty < x < \infty$$

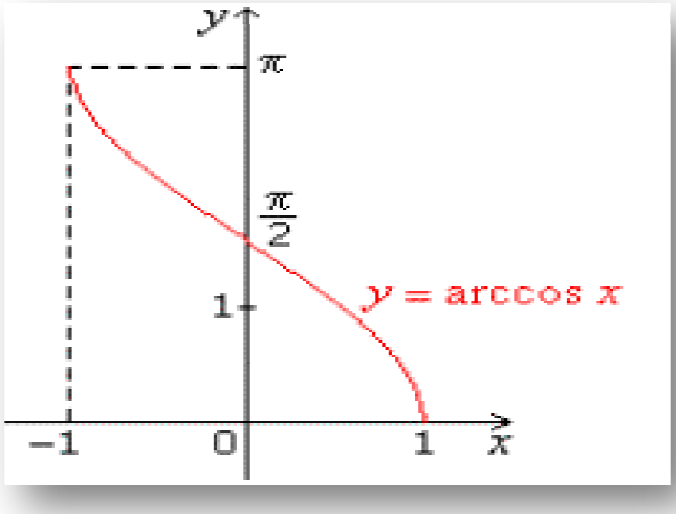
المشتق

$$\frac{dy}{dx} = \frac{-1}{\sqrt[2]{1-x^2}}$$

الرسم البياني

Arccos(x): [-1;1] ➔ *[0; π]*

X ➔ arccos (x)



Y=Tan(x) الدالة

النشر المحدود للدالة

$$\tan(x)= x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + o(x^8), \quad -\infty < x < \infty$$

الشكل الأسي

$$\tan(x)= -i\left(\frac{e^{ix}-e^{-ix}}{e^{ix}+e^{-ix}}\right)$$

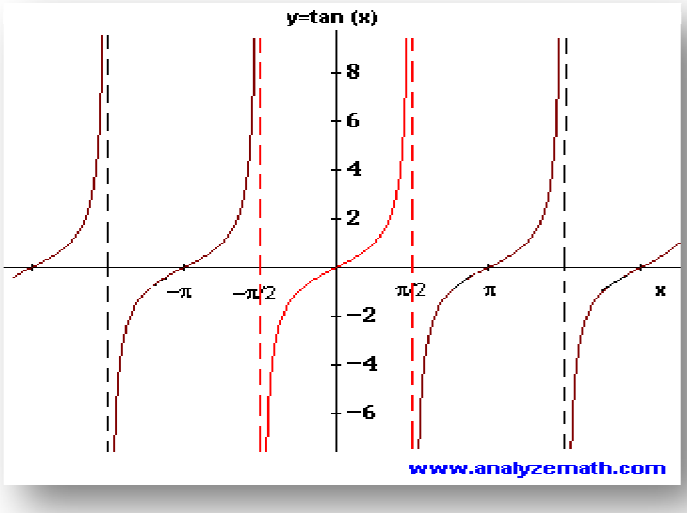
المشتق

$$\frac{dy}{dx} = \frac{1}{\cos^2(x)} = 1 + \tan^2(x)$$

الرسم البياني

Tan(x):] $-\pi/2;\pi/2[$ ➔|R

X ➔ tan (x)



بعض العلاقات التي تخص الدالة tan(x)

$$tan^2(\alpha) = \frac{sin^2(\alpha)}{cos^2(\alpha)}$$

$$= -1 + \frac{1}{cos^2(\alpha)}$$

$$= \frac{1 - cos(2\alpha)}{1 + cos(2\alpha)}$$

$$= \frac{\sin^2(\alpha)}{1-\sin^2(\alpha)}$$

$\tan(0) = 0$

$\tan(-\theta) = -\tan(\theta)$ ... ... (fonction impair)

$$tan(\alpha \pm \beta) = \frac{tan(\alpha) \pm tan(\beta)}{1 \mp tan(\alpha)tan(\beta}$$

$$tan(2\alpha) = \frac{2tan(\alpha)}{1 - tan^2(\alpha)}$$

$$tan(\pi \mp \theta) = \frac{sin(\pi \mp \theta)}{cos(\pi \mp \theta)} = \frac{\pm sin(\theta)}{-cos(\theta)} = \mp tan(\theta)$$

$$\tan\left(\frac{\pi}{2} \pm \theta\right) = \frac{sin(\frac{\pi}{2} \pm \theta)}{\cos(\frac{\pi}{2} \pm \theta)} = \frac{\cos(\theta)}{\mp \sin(\theta)} = \mp \frac{1}{\tan(\theta)} = \mp \cot(\theta)$$

$$tan(\alpha) \pm tan(\beta) = \frac{Sin(\alpha \pm \beta)}{cos(\alpha)cos(\beta)}$$

$Y=\tan^{-1}(x)=Arctan(x)$ دالتها العكسية هي

زوارق يحي

ZOUAREG YAHIA

النشر المحدود للدالة

$$\text{Arctan(x)} = \sum_{n\geq 0} \frac{(-1)^n x^{2n+1}}{2n+1} = x - \frac{x^3}{3} + \cdots + \frac{(-1)^p x^{2p+1}}{2p+1} + o(x^{2p+2}),\ -\infty < x < \infty$$

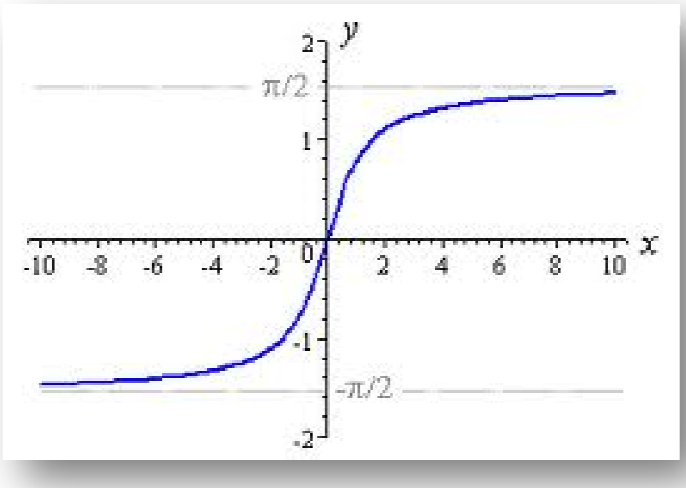
المشتق

$$\frac{dy}{dx} = \frac{1}{1+x^2}$$

الرسم البياني

Arctan(x): |R ➔] $-\pi/2;\pi/2$[

X ➔ Arc tan (x)



الدالة y =cotan(x)

النشر المحدود للدالة

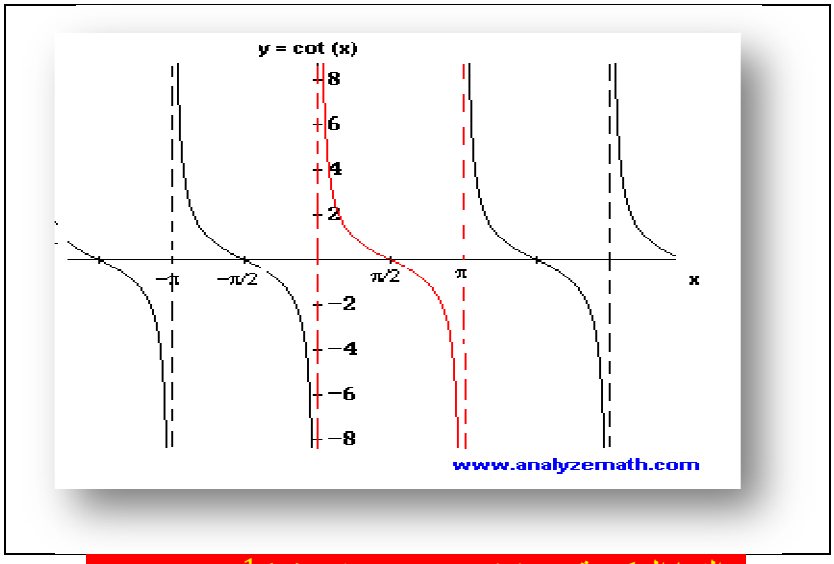
المشتق

$$\frac{dy}{dx} = \frac{-1}{\sin^2(x)} = -1 - \cot\tan^2(x)$$

الرسم البياني

cotan(x):]0;π [➔ℝ

X ➔ cotan (x)



دالتها العكسية هي Y=cotan$^{-1}(x)$=Arccotang(x)

النشر المحدود للدالة

$$\text{Arccotan(x)} = \frac{\pi}{2} + \sum_{n\geq 1} \frac{(-1)^n x^{2n+1}}{2n+1} = \frac{\pi}{2} - x + \frac{x^3}{3} - \cdots + \frac{(-1)^p x^{2p+1}}{2p+1} + o(x^{2p+2}),$$
$$-\infty < x < \infty$$

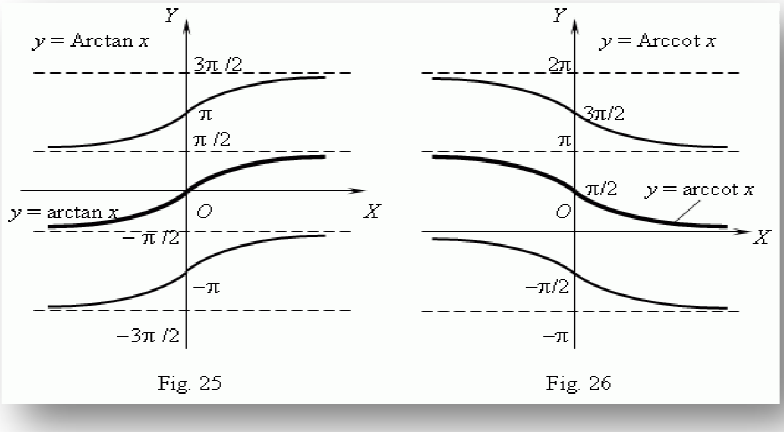
المشتق

$$\frac{dy}{dx} = \frac{-1}{1+x^2}$$

الرسم البياني

Arccotan(x): ℝ➔]0;π [

X ➔ Arccotan (x)



الدالة Y=ch(x)

النشر المحدود للدالة

$$\text{ch}(x) = \sum_{n\geq 0} \frac{x^{2n}}{2n!} = 1 + \frac{x^2}{2!} + \cdots + \frac{x^{2p}}{2p!} + o(x^{2p+1}),\ -\infty < x < \infty$$

الشكل الأسي

$$\text{ch}(x) = \frac{e^x + e^{-x}}{2}$$

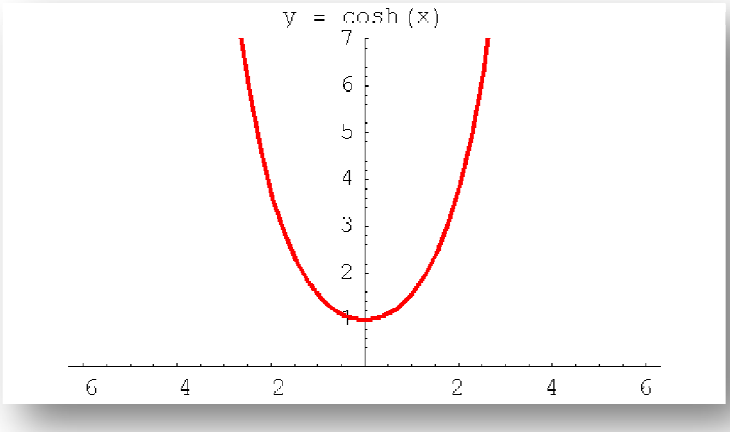
المشتق

$$\frac{dy}{dx} = sh(x)$$

الرسم البياني

ch(x): ℝ➔[1;+∞[

X ➔ ch (x)



بعض العلاقات التي تخص ch

$ch(0) = 1$

$\text{ch}(-\theta) = \text{ch}(\theta)$ ... ... (fonction pair)

$ch^2(\alpha) = 1 + sh^2(\alpha)$

$ch(\alpha \pm \beta) = ch(\alpha)ch(\beta) \pm sh(\alpha)sh(\beta)$

$ch\ (2\alpha) = ch^2(\alpha) + sh^2(\alpha)$

$= 2ch^2(\alpha) + 1$

$= 2\,sh^2(\alpha) - 1$

$= \frac{1 + th^2(\alpha)}{1 - th^2(\alpha)}$

$ch(\alpha) + ch(\beta) = 2ch(\frac{\alpha+\beta}{2})ch(\frac{\alpha-\beta}{2})$

$ch(\alpha) - ch(\beta) = 2sh(\frac{\alpha+\beta}{2})sh(\frac{\alpha-\beta}{2})$

بعض العلاقات التي تربطهما sh و ch

$$\mathrm{ch}^2(\alpha) - \mathrm{sh}^2(\alpha) = 1$$
$$\mathrm{ch}(\alpha) \pm \mathrm{sh}(\alpha) = \mathrm{e}^{\pm\alpha}$$
$$[ch(\alpha) \pm sh(\alpha)]^n = ch(n\alpha) \pm sh(n\alpha) = e^{\pm n\alpha} = (e^{\pm\alpha})^n$$

دالتها العكسية هي Argch(x)=$Y=\mathrm{ch}^{-1}(x)$

النشر المحدود للدالة

المشتق

$$\frac{dy}{dx} = \frac{1}{\sqrt[2]{x^2-1}}$$

الدالة sh(x)= y

النشر المحدود للدالة

$$\mathrm{sh}(x) = \sum_{n\geq 1} \frac{x^{2n-1}}{(2n-1)!} = x + \frac{x^3}{3!} + \cdots + \frac{x^{2p-1}}{(2p-1)!} + o(x^{2p}),\ -\infty < x < \infty$$

الشكل الأسي

$$\mathrm{sh}(x) = \frac{e^x - e^{-x}}{2}$$

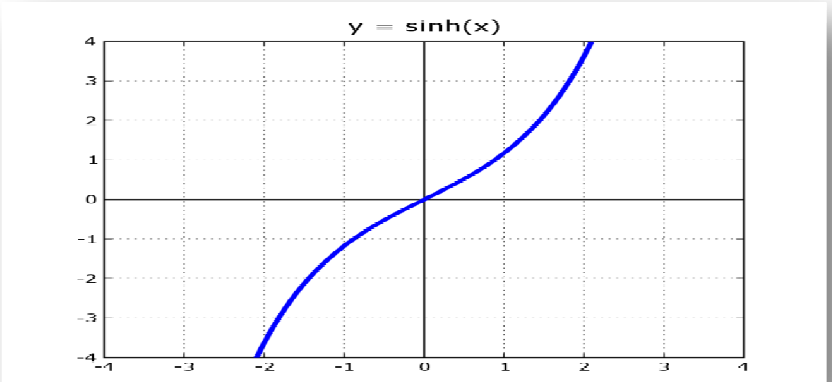
المشتق

$$\frac{dy}{dx} = ch(x)$$

الرسم البياني

sh(x): ℝ ➔ ℝ

X ➔ sh (x)



بعض العلاقات التي تخص sh

$sh(0) = 0$

$\mathrm{sh}(-\theta) = -\mathrm{sh}(\theta)$ ... ... (fonction impair)

$sh^2(\alpha) = ch^2(\alpha) - 1$

$sh(\alpha \pm \beta) = sh(\alpha)ch(\beta) \pm sh(\beta)ch(\alpha)$

$sh(2\alpha) = 2sh(\alpha)ch(\alpha)$

$= \frac{2\,th(\alpha)}{1-th^2(\alpha)}$

$sh(\alpha) \pm sh(\beta) = 2sh(\frac{\alpha \pm \beta}{2})ch(\frac{\alpha \mp \beta}{2})$

دالتها العكسية هي Y=sh$^{-1}(x)$=Argsh(x)

النشر المحدود للدالة

$$\mathrm{sh}(x) = \sum_{n\geq 1} \frac{x^{2n-1}}{(2n-1)!} = x + \frac{x^3}{3!} + \cdots + \frac{x^{2p-1}}{(2p-1)!} + o(x^{2p}),\ -\infty < x < \infty?$$

المشتق

$$\frac{dy}{dx} = \frac{1}{\sqrt[2]{1+x^2}}$$

الدالة y =th(x)

النشر المحدود للدالة

$$\mathrm{Th(x)} = x - \frac{x^3}{3} + \frac{2x^5}{15} - \frac{17x^7}{315} + o(x^8),\ -\infty < x < \infty$$

الشكل الأسي

$$\mathrm{th(x)} = \frac{\mathrm{sh}(x)}{\mathrm{ch}(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

المشتق

$$\frac{dy}{dx} = \frac{4}{(e^x+e^{-x})^2} = [\frac{2}{(e^x+e^{-x})}]^2 = (\frac{1}{ch(x)})^2 = \frac{1}{\mathrm{ch}^2(\mathrm{x})}$$

الرسم البياني

th(x): ℝ ➔]-1;1[

X ➔ th (x)

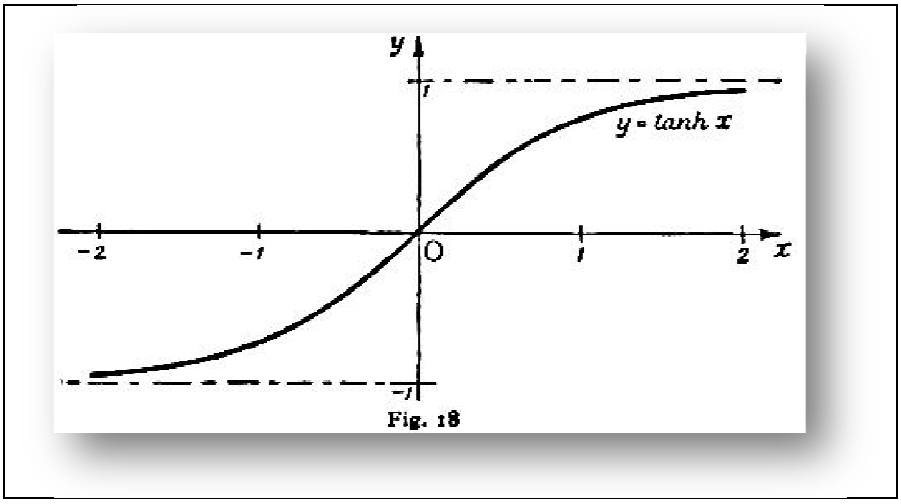


Fig. 18

بعض العلاقات التي تخص th

$th(0) = 0$

$\text{th}(-\theta) = -\text{th}(\theta)$ ..... (fonction impair)

$$th^2(\alpha) = \frac{sh^2(\alpha)}{ch^2(\alpha)} = 1 - \frac{1}{cos^2(\alpha)} = -\frac{1 - ch(2\alpha)}{1 + ch(2\alpha)} = \frac{\text{sh}^2(\alpha)}{1+\text{sh}^2(\alpha)}$$

$$th(\alpha \pm \beta) = \frac{th(\alpha) \pm th(\beta)}{1 \pm th(\alpha)th(\beta)}$$

$$th(2\alpha) = \frac{2th(\alpha)}{1 + th^2(\alpha)}$$

$$th(\alpha) \pm th(\beta) = \frac{Sh(\alpha \pm \beta)}{ch(\alpha)ch(\beta)}$$

دالتها العكسية هي $Y=\text{th}^{-1}(x)=\text{Argth}(x)$

النشر المحدود للدالة

$$\text{Argth}(x) = \sum_{n\geq 1} \frac{x^{2n+1}}{(2n+1)} = x + \frac{x^3}{3} + \cdots + \frac{x^{2p+1}}{(2p+1)} + o(x^{2p+2}),\ -\infty < x < \infty$$

$$\text{Argth(x)} = -\frac{1}{2} ln\left(\frac{1+x}{1-x}\right) = \frac{1}{2} ln\left(\frac{1-x}{1+x}\right)$$

المشتق

$$\frac{dy}{dx} = \frac{-1}{1 - x^2}$$

الدالة $y = coth(x)$

النشر المحدود للدالة

الشكل الأسي

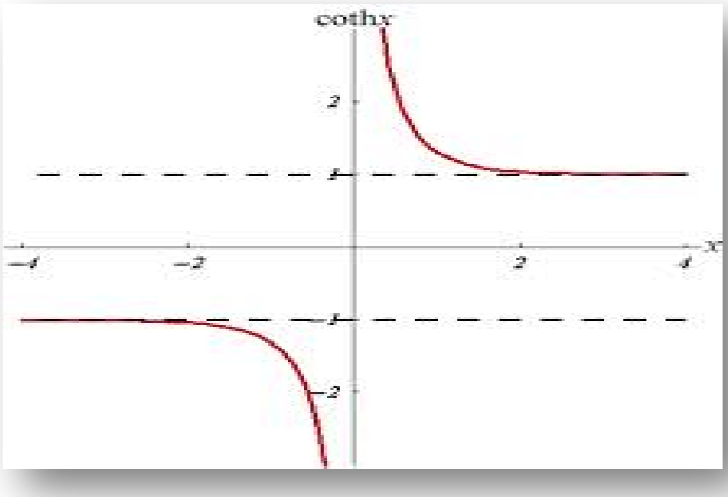
المشتق

$$\frac{dy}{dx} = \frac{-4}{(e^x - e^{-x})^2} = -[\frac{2}{(e^x - e^{-x})}]^2 = -(\frac{1}{sh(x)})^2 = \frac{-1}{\text{sh}^2(\text{x})}$$

الرسم البياني

coth(x): $\mathbb{R}\backslash\{0\}$ ➔ $\mathbb{R}\backslash\{[-1\ ;1]\}$

X ➔ coth (x)



دالتها العكسية هي $Y=\coth^{-1}(x)=\text{Argcoth}(x)$

النشر المحدود للدالة

?

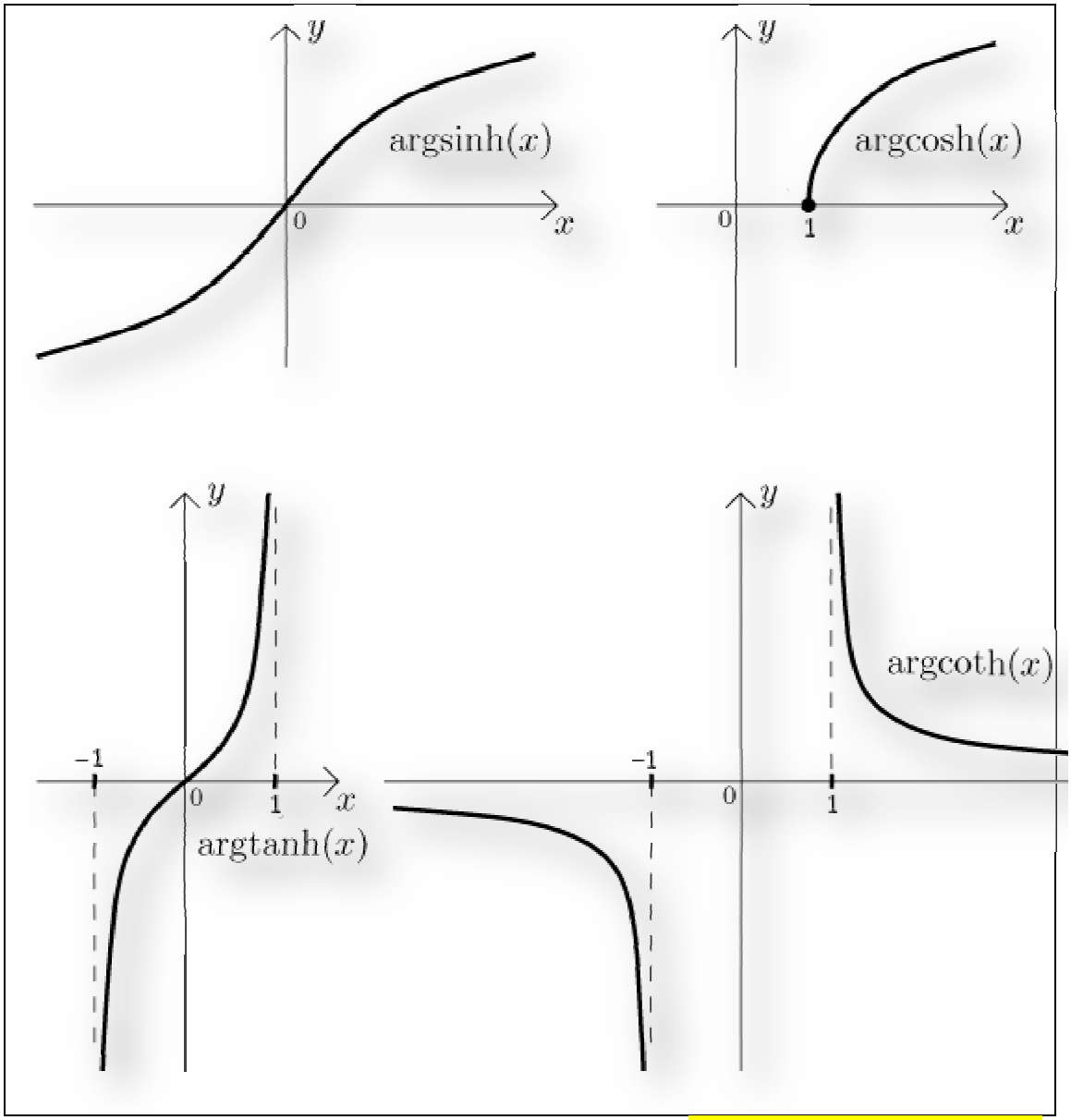
$$\text{Argcoth(x)}=\frac{1}{2}\,ln(\frac{1+x}{1-x})$$

المشتق

$$\frac{dy}{dx} = \frac{1}{1-x^2}$$

الرسم البياني

Argcosh :	[1;+∞[➔ $\mathbb{R}$
Argsinh:	$\mathbb{R}$ ➔ $\mathbb{R}$
Argtanh:	]-1;1[➔ $\mathbb{R}$
Argcoth:	$\mathbb{R}\backslash\{[-1\ ;1]\}$ ➔ $\mathbb{R}\backslash\{0\}$



جدول يوضح دوال وبعض مشتقاتها

الدالة	مجموعة الانطلاق	مجموعة الوصول	مشتقتها
Cos(x)	[0; π]	[-1;1]	-Sin(x)
Arccos(x)	[-1;1]	[0; π]	$\frac{-1}{\sqrt{1-x^2}}$
Ch(x)	ℝ	[1;+∞[	Sh(x)
Argch(x)	[1;+∞[	ℝ	$\frac{1}{\sqrt{x^2-1}}$
Sin(x)	[-π/2;π/2]	[-1 ;1]	Cos(x)
Arcsin(x)	[-1 ;1]	[-π/2;π/2]	$\frac{1}{\sqrt{1-x^2}}$
Sh(x)	ℝ	ℝ	Ch(x)
Argsh(x)	ℝ	ℝ	$\frac{1}{\sqrt{x^2+1}}$

$\tan x = \frac{\sin x}{\cos x}$	] $-\pi/2;\pi/2$[	$\mathbb{R}$	$1+tan^2(x)=\frac{1}{cos^2}$
Arctang(x)	$\mathbb{R}$	] $-\pi/2;\pi/2$[	$\frac{1}{1+x^2}$
th(x)= $\frac{sh\,x}{ch\,x}$	$\mathbb{R}$	]-1;1[	$\frac{1}{ch^2} = \frac{4}{(e^x+e^{-x})^2}$
Argth(x)	]-1;1[	$\mathbb{R}$	$\frac{-1}{1-x^2}$
Cotang(x)	]0;π [	$\mathbb{R}$	$-1-cotang^2(x)=\frac{-1}{sin^2}$
Arccotang(x)	$\mathbb{R}$	]0;π [	$\frac{-1}{1+x^2}$
Coth(x)	$\mathbb{R}\backslash\{0\}$	$\mathbb{R}\backslash\{[-1;1]\}$	$\frac{-1}{sh^2} = \frac{-4}{(e^x-e^{-x})^2}$
Argcoth(x)	$\mathbb{R}\backslash\{[-1;1]\}$	$\mathbb{R}\backslash\{0\}$	$\frac{1}{1-x^2}$