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Validation of Dynamic Relaxation (DR) Method in Rectangular Laminates using Large Deflection Theory

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Abstract— First order orthotropic shear deformation equations for the nonlinear elastic bending response of rectangular plates are introduced. Their solution using a computer program based on finite differences implementation of the Dynamic Relaxation (DR) method is outlined. The convergence and accuracy of the DR solutions for elastic large deflection response of isotropic, orthotropic, and laminated plates are established by comparison with various exact and approximate solutions. The present Dynamic Relaxation method (DR) coupled with finite differences method shows a fairly good agreement with other analytical and numerical methods used in the verification scheme.It was found that: The convergence and accuracy of the DR solution is dependent on several factors including boundary conditions, mesh size and type, fictitious densities, damping coefficients, time increment and applied load. Also, the DR large deflection program using uniform finite differences meshes can be employed in the analysis of different thicknesses for isotropic, orthotropic or laminated plates under uniform loads. All the comparison results for simply supported (SS4) edge conditions showed that deflection is almost dependent on the direction of the applied load or the arrangement of the layers

Keywords— Dynamic relaxation, rectangular laminates, large deflection theory, isotropic, orthotropic, laminated plates.

Notations

a, b plate side lengths in x and y directions respectively.
$A_{ij}\ (i,j = 1,2,6)$ Plate in plane stiffness.
A_{44}, A_{55} Plate transverse shear stiffness.
$D_{ij}\ (i,j = 1,2,6)$ Plate flexural stiffness.
$\varepsilon_x^o, \varepsilon_y^o, \varepsilon_{xy}^o$ Mid – plane direct and shear strains
$\varepsilon_{xz}^o, \varepsilon_{yz}^o$ Mid – plane transverse shear strains.
E_1, E_2, G_{12} In – plane elastic longitudinal, transverse and shear moduli.
G_{13}, G_{23} Transverse shear moduli in the x – z and y – z planes respectively.
M_x, M_y, M_{xy} Stress couples.
$\bar{M}_x, \bar{M}_y, \bar{M}_{xy}$ Dimensionless stress couples.
N_x, N_y, N_{xy} Stress resultants.
$\bar{N}_x, \bar{N}_y, \bar{N}_{xy}$ Dimensionless stress resultants.
$\bar{q}$ Dimensionless transverse pressure.
Q_x, Q_y Transverse shear resultants.
u, v In – plane displacements.
w Deflections
$\bar{w}$ Dimensionless deflection
x, y, z Cartesian co – ordinates.
δt Time increment
$\emptyset, \psi$ Rotations of the normal to the plate mid – plane
ν_{xy} Poisson's ratio
$\ell_u, \ell_v, \ell_w, \ell_\emptyset, \ell_\psi$ In plane, out of plane and rotational fictitious densities.
$\chi_x^o, \chi_y^o, \chi_{xz}^o$ Curvature and twist components of plate mid – plane

I. INTRODUCTION

Composites were first considered as structural materials a little more than half a century ago. From that time to now, they have received increasing attention in all aspects of material science, manufacturing technology, and theoretical analysis.

The term composite could mean almost anything if taken at face value, since all materials are composites of dissimilar subunits if examined at close enough details. But in modern engineering materials, the term usually refers to a matrix material that is reinforced with fibers. For instance, the term "FRP" which refers to Fiber Reinforced plastic, usually indicates a thermosetting polyester matrix containing glass fibers, and this particular composite has the lion's share of today commercial market.

In the present work, a numerical method known as Dynamic Relaxation (DR) coupled with finite differences is used. The DR method was first proposed in 1960s and then passed through a series of studies to verify its validity by Turvey and Osman [4], [8] and [9] and Rushton [2], Cassel and Hobbs [10], and Day [11]. In this method, the equations of equilibrium are converted to dynamic equations by adding damping and inertia terms. These are then expressed in finite difference form and the solution is obtained through iterations. The optimum damping coefficient and time increment used to stabilize the solution depend on a number of factors including the matrix properties of the structure, the applied load, the boundary conditions and the size of the mesh used.

Numerical techniques other than the DR include finite element method, which widely used in the present studies i.e. of Damodar R. Ambur et al [12], Ying Qing Huang et al [13], Onsy L. Roufaeil et al [14]… etc. In a comparison between the DR and the finite element method, Aalami [15] found that the computer time required for finite element method is eight times greater than for DR analysis, whereas the storage capacity for finite element analysis is ten times or more than that for DR analysis. This fact is supported by Putcha and Reddy [16] who noted that some of the finite element formulations require large storage capacity and computer time. Hence, due to less computations and computer time involved in the present study. The DR method is considered more efficient than the finite element method. In another comparison Aalami [15] found that the difference in accuracy between one version of finite element and another may reach a value of 10% or more, whereas a comparison between one version of finite element method and DR showed a difference of more than 15%. Therefore, the DR method can be considered of acceptable accuracy. The only apparent limitation of DR method is that it can only be applied to limited geometries. However, this limitation is irrelevant to rectangular plates which are widely used in engineering applications.

The Dynamic Relaxation (DR) program used in this paper is designed for the analysis of rectangular plates irrespective of material, geometry, edge conditions. The functions of the program are to read the file data; compute the stiffness of the laminate, the fictitious densities, the velocities and displacements and the mid – plane deflections and stresses; check the stability of the numerical computations, the convergence of the solution, and the wrong convergence; compute through – thickness stresses in direction of plate axes; and transform through – thickness stresses in the lamina principal axes.

The convergence of the DR solution is checked at the end of each iteration by comparing the velocities over the plate domain with a predetermined value which ranges between 10^{-9} for small deflections and 10^{-6} for large deflections. When all velocities are smaller than a predetermined value, the solution is deemed converged and consequently the iterative procedure is terminated. Sometimes DR solution converges to an invalid solution. To check for that the profile of the variable is compared with an expected profile over the domain. For example, when the value of the function on the boundaries is zero, and it is expected to increase from edge to center, then the solution should follow a similar profile. When the computed profile is different from the expected values, the solution is considered incorrect and can hardly be made to converge to the correct value by altering the damping coefficients and time increment. Therefore, the boundary conditions should be examined and corrected if they are improper.

The errors inherent in the DR technique ([17] – [27]) include discretization error which is due to the replacement of a continuous function with a discrete function, and there is an additional error because the discrete equations are not solved exactly due to the variations of the velocities from the edge of the plate to the center. Finer meshes reduce the discretization error, but increase the round – off error due to the large number of calculations involved.

II. LARGE DEFLECTION THEORY

The equilibrium, strain, constitutive equations and boundary conditions are introduced below without derivation

2.1 Equilibrium equations:

$$\frac{\partial N_x}{\partial x}+\frac{\partial N_{xy}}{\partial y}=0$$

$$\frac{\partial N_{xy}}{\partial x}+\frac{\partial N_y}{\partial y}=0$$

$$N_x\frac{\partial^2 w}{\partial x^2}+2N_{xy}\frac{\partial^2 w}{\partial x\partial y}+N_y\frac{\partial^2 w}{\partial y^2}+\frac{\partial Q_x}{\partial x}+\frac{\partial Q_y}{\partial y}+q=0 \qquad (1)$$

$$\frac{\partial M_x}{\partial x}+\frac{\partial M_{xy}}{\partial y}-Q_x=0$$

$$\frac{\partial M_{xy}}{\partial x}+\frac{\partial M_y}{\partial y}-Q_y=0$$

2.2 Strain equations

The large deflection strains of the mid – plane of the plate are as given below:

$$\varepsilon_x^\circ=\frac{\partial u^\circ}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2+z\frac{\partial \phi}{\partial x}$$

$$\varepsilon_y^{\circ} = \frac{\partial v^{\circ}}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 + z\frac{\partial \psi}{\partial y}$$

$$\varepsilon_{xy}^{\circ} = \frac{\partial u^{\circ}}{\partial y} + \frac{\partial v^{\circ}}{\partial x} + \frac{\partial w}{\partial x} - \frac{\partial w}{\partial y} + z\left(\frac{\partial \phi}{\partial y} + \frac{\partial \psi}{\partial x}\right) \qquad (2)$$

$$\varepsilon_{xz}^{\circ} = \frac{\partial w}{\partial y} + \psi$$

$$\varepsilon_{yz}^{\circ} = \frac{\partial w}{\partial x} + \phi$$

2.3 The constitutive equations

The laminate constitutive equations can be represented in the following form:

$$\begin{Bmatrix} N_i \\ M_i \end{Bmatrix} = \begin{bmatrix} A_{ij} & B_{ij} \\ B_{ij} & D_{ij} \end{bmatrix} \begin{Bmatrix} \varepsilon_j^o \\ \chi_j^o \end{Bmatrix} \qquad (3)$$

$$\begin{Bmatrix} N_i \\ M_i \end{Bmatrix} = \begin{bmatrix} A_{ij} & B_{ij} \\ B_{ij} & D_{ij} \end{bmatrix} \begin{Bmatrix} \varepsilon_j^o \\ \chi_j^o \end{Bmatrix}$$

WhereN_i refers to N_x ,N_y and,N_{xy} and $M_i$denotesM_x,$M_y$andM_{xy}.A_{ij},B_{ij} and D_{ij} *(i, j=1, 2, 6)* are respectively the membrane rigidities, coupling rigidities and flexural rigidities of the plate.χ_j^o Denotes$\frac{\partial \phi}{\partial x}$, $\frac{\partial \psi}{\partial y}and\frac{\partial \phi}{\partial y} + \frac{\partial \psi}{\partial x}$.$A_{44}$, $A_{45}$and A_{55} denote the stiffness Coefficients and are calculated as follows:-

$$A_{ij} = \sum_{k-1}^{n} k_i k_j \int_{z_k}^{z_{k+1}} C_{ij}\, dz, (i, j = 4, 5)$$

WhereC_{ij} the stiffness of a lamina is referred to the plate principal axes, andK_i, $K_j$are the shear correction factors.

2.4 Boundary conditions

Four sets of simply supported boundary conditions are used in this paper, and are denoted as SS1, SS2, SS3, and SS4 as has been shown in Fig. (1) Below:

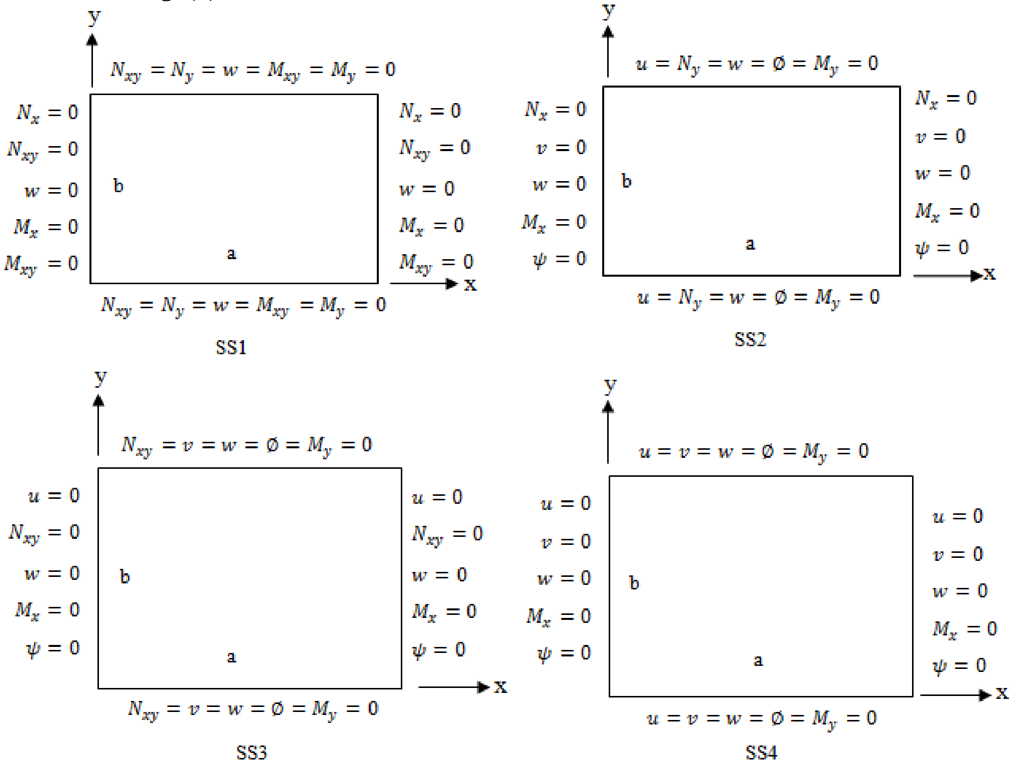

Fig. (1) Simply supported boundary conditions

III. DYNAMIC RELAXATION OF THE PLATE EQUATIONS

An exact solution of the plate equations is obtained using finite differences coupled with dynamic relaxation method. The damping and inertia terms are added to equations (1). Then the following approximations are introduced for the velocity and acceleration terms:

$$\frac{\partial \alpha}{\partial t} = \frac{1}{2}\left[\frac{\partial \alpha^a}{\partial t} + \frac{\partial \alpha^b}{\partial t}\right]$$

(4)

$$\frac{\partial^2 \alpha}{\partial t^2} = \left(\frac{\partial \alpha^a}{\partial t} - \frac{\partial \alpha^b}{\partial t}\right) / \partial t$$

In which$\alpha \equiv u, v, w, \phi, \psi$. Hence equations (1) become:

$$\frac{\partial u^a}{\partial t} = \frac{1}{1+k_u^*}\left[\left(1-k_u^*\right)\frac{\partial u^b}{\partial t} + \frac{\delta t}{\ell u}\left(\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y}\right)\right]$$

$$\frac{\partial v^a}{\partial t} = \frac{1}{1+k_v^*}\left[\left(1-k_v^*\right)\frac{\partial v^b}{\partial t} + \frac{\delta t}{\ell u}\left(\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y}\right)\right]$$

$$\frac{\partial w^a}{\partial t} = \frac{1}{1+k_w^*}\left[\left(1-k_w^*\right)\frac{\partial w^b}{\partial t} + \frac{\delta t}{\ell_w}\left(N_x\frac{\partial^2 w}{\partial x^2} + 2N_{xy}\frac{\partial^2 w}{\partial x \partial y} + N_y\frac{\partial^2 w}{\partial y^2} + \frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + q\right)\right]$$

$$\frac{\partial \phi^a}{\partial t} = \frac{1}{1+k_\phi^*}\left[\left(1-k_\phi^*\right)\frac{\partial \phi^b}{\partial t} + \frac{\delta t}{\ell_\phi}\left(\frac{\partial M_y}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x\right)\right]$$

$$\frac{\partial \psi^a}{\partial t} = \frac{1}{1+k_\psi^*}\left[\left(1-k_\psi^*\right)\frac{\partial \psi^b}{\partial t} + \frac{\delta t}{\ell_\psi}\left(\frac{\partial N_{xy}}{\partial x} + \frac{\partial M_y}{\partial y} - Q_y\right)\right] \quad (5)$$

The superscripts a and b in equations (4) and (5) refer respectively to the values of velocities after and before the time incrementδt and$k_\alpha^* = \frac{1}{2}k_\alpha \delta t \ell_\alpha^{-1}$. The displacements at the end of each time increment,δt, are evaluated using the following integration procedure:

$$\alpha^a = \alpha^b + \delta t \frac{\partial \alpha^b}{\partial t} \quad (6)$$

Thus equations (5), (6), (2) and (3) constitute the set of equations for the dynamic relaxation solution. The DR procedure operates as follows:

(1) Set initial conditions.
(2) Compute velocities from equations (5).
(3) Compute displacement from equation (6).
(4) Apply displacement boundary conditions.
(5) Compute strains from equations (2).
(6) Compute stress resultants and stress couples from equations (3).
(7) Apply stress resultants and stress couples boundary conditions.
(8) Check if velocities are acceptably small (say 10^{-6}).
(9) Check if the convergence criterion is satisfied, if it is not repeat the steps from 2 to 8.

It is obvious that this method requires five fictitious densities and a similar number of damping coefficients so as the solution will be converged correctly.

IV. RESULTS AND DISCUSSIONS

Various verification exercises of the dynamic relaxation (DR) method using large deflection theory were undertaken including isotropic, orthotropic and laminated rectangular plates as shown below:

Table 1 shows deflections, stress resultants and stress couples in simply supported in – plane free (SS2) isotropic plate. The present results have been computed with6×6 uniform meshes. These results are in a fairly good agreement with those of Aalami et al [1] using finite difference method (i.e. for deflections, the difference ranges between 0.35% at $\bar{q} = 20.8$ and 0 % as the pressure is increased to 97). A similar comparison between the two results is shown in Table 2 for simply supported (SS3) condition. It is apparent that the center deflections, stress couples and stress resultants agree very well. The mid – side stress resultants do not show similar agreement whilst the corner stress resultants show considerable differences. This may be attributed to the type of mesh used in each analysis. A set of thin plate results comparisons presented here with Rushton [2] who employed the DR method coupled with finite differences. The present results for simply supported (SS4) square plates were computed for two thickness ratios using a 8×8 uniform mesh are listed in Table 3. In this instant, the present results differ slightly from those found in [2]. Another comparison for simply supported (SS4) square isotropic plates subjected to uniformly distributed loads are shown in Tables 4 and 5 respectively for deflection analysis of thin and moderately thick plates. In this comparison, it is noted that, the center deflection of the present DR analysis, and those of Azizian and Dawe [3] who employed the finite strip method are in fairly good agreement (i.e. with a maximum error not exceeding 0.09%).

A large deflection comparison for orthotropic plates was made with the DR program. The results are compared with DR results of Turvey and Osman [4], Reddy's [5], and Zaghloul et al results [6]. For a thin uniformly loaded square plate made of material I which its properties are stated in Table 6 and with simply supported in – plane free (SS2) edges. The center deflections are presented in Table 7 where DR showed a good agreement with the other three.

A large deflection comparison for laminated plates was made by recomposing sun and chin's results [7] for $[90_4^o/0_4^0]$ using the DR program and material II which its properties are cited in Table 6. The results were obtained for quarter of a plate using a 5×5 square mesh, with shear correction factors$k_4^2 = k_5^2 = 5/6$. The analysis was made for different boundary conditions and the results were shown in Tables 8, and 9 as follows: The present DR deflections of two layer anti–symmetric cross – ply simply supported in – plane fixed (SS4) are compared with DR results of Turvey and Osman [8] and with sun and chin's [7] values for a range of loads as shown in Table 8. The good agreement found confirms that for simply supported (SS4) edge conditions, the deflection depends on the direction of the applied load or the arrangement of the layers. Table 9 shows a comparison between the present DR, and DR [8] results, which are approximately identical. The difference between laminates$[0^o/90^o]$ and$[90^o/0^o]$ at$b/a = 5$is 0.3% whilst it is 0% when$b/a = 1$.

The comparisons made between DR and alterative techniques show a good agreement and hence the present DR large deflection program using uniform finite difference meshes can be employed with confidence in the analysis of moderately thick and thin flat isotropic, orthotropic and laminated plates under uniform loads. The program can be used with the same confidence to generate small deflection results.

Table 1 comparison of present DR, Aalami and Chapman's [1] large deflection results for simply supported (SS2) square isotropic plate subjected to uniform pressure $(h/a = 0.02, \nu = 0.3)$

$\bar{q}$	S	$\bar{w}_c$	$\bar{M}_x(1)$ $\bar{M}_y(2)$	$\bar{N}_x(1)$ $\bar{N}_y(2)$
20.8	1 2	0.7360 0.7386	0.7357 0.7454	0.7852 0.8278
41.6	1 2	1.1477 1.1507	1.0742 1.0779	1.8436 1.9597
63.7	1 2	1.4467 1.4499	1.2845 1.2746	2.8461 3.0403
97.0	1 2	1.7800 1.7800	1.4915 1.4575	4.1688 4.4322

S (1): present DR results (6×6uniform mesh over quarter of the plate)
S (2): [1] results (6×6graded mesh over quarter of the plate)
$(1)\ x = y = \frac{1}{2}a, z = 0$

Table 2 Comparison of present DR, Aalami and Chapman's [1] large deflection results for simply supported (SS3) square isotropic plate subjected to uniform pressure $(h/a = 0.02, \nu = 0.3)$

$\bar{q}$	S	$\bar{w}_c$	$\bar{M}_x(1)$ $\bar{M}_y(1)$	$\bar{N}_x(1)$ $\bar{N}_y(1)$	$\bar{N}_x(2)$ $\bar{N}_y(3)$	$\bar{N}_x(3)$ $\bar{N}_y(2)$	$\bar{N}_x(4)$ $\bar{N}_y(4)$
20.8	1 2	0.5994 0.6094	0.6077 0.6234	1.0775 1.0714	0.2423 0.2097	1.1411 1.1172	0.1648 0.2225
41.6	1 2	0.8613 0.8783	0.8418 0.8562	2.2435 2.2711	0.5405 0.4808	2.4122 2.4084	0.3177 0.4551
63.7	1 2	1.0434 1.0572	0.9930 1.0114	3.3151 3.3700	0.8393 0.7564	3.6014 3.6172	0.4380 0.6538
97.0	1 2	1.2411 1.2454	1.1489 1.1454	4.7267 4.8626	1.2604 1.1538	5.1874 2.2747	0.5706 0.9075

S (1): present DR results (6×6uniform mesh over quarter of the plate)
S (2): [1] results (6×6graded mesh over quarter of the plate)
$(1)\ x = y = \frac{1}{2}a, z = 0; (2) x = \frac{1}{2}a, y = z = 0; (3) x = 0, y = \frac{1}{2}a, z = 0; (4) x = y = z = 0$

Table 3 Comparison of present DR, and Rushton's [2] large deflection results for simply supported (SS4) square isotropic plate subjected to uniform pressure $(\nu = 0.3)$

$\bar{q}$	S	$\bar{w}_c$	$\bar{\sigma}_1(1)$
8.2	1 2 3	0.3172 0.3176 0.2910	2.3063 2.3136 2.0900
29.3	1 2 3	0.7252 0.7249 0.7310	5.9556 5.9580 6.2500
91.6	1 2 3	1.2147 1.2147 1.2200	11.3180 11.3249 11.4300

293.0	1 2 3	1.8754 1.8755 1.8700	20.749 20.752 20.820

S (1): present DR results ($h/a = 0.02; 8 \times 8$uniform mesh over quarter of the plate)
S (2): present DR results ($h/a = 0.01; 8 \times 8$uniform mesh over quarter of the plate)
S (3): [2] results (thin plate 8×8 uniform mesh over quarter of the plate)
(1) $x = y = \frac{1}{2}a, z = \frac{1}{2}h$

Table 4 Comparison of the present DR, and Azizian and Dawe's [3] large deflection results for thin shear deformable simply supported (SS4) square isotropic plate subjected to uniform pressure ($h/a = 0.01, \nu = 0.3$)

$\bar{q}$	S	$\bar{w}_c$
9.2	1 2	0.34693 0.34677
36.6	1 2	0.80838 0.81539
146.5	1 2	1.45232 1.46250
586.1	1 2	2.38616 2.38820

S (1): present DR results (6×6uniform mesh over quarter of the plate)
S (2): Azizian and Dawe [3] results.

Table 5 Comparison of the present DR, and Azizian and Dawe's [3] large deflection results for moderately thick shear deformable simply supported (SS4) square isotropic plates subjected to uniform pressure ($h/a = 0.05, \nu = 0.3$)

$\bar{q}$	S	$\bar{w}_c$
0.92	1 2	0.04106 0.04105
4.6	1 2	0.19493 0.19503
6.9	1 2	0.27718 0.27760
9.2	1 2	0.34850 0.34938

S (1): present DR results (6×6uniform mesh over quarter of the plate)
S (2): Azizian and Dawe [3] results.

Table 6 Material properties used in orthotropic and laminated plate comparison analysis.

Material	E_1/E_2	G_2/E_2	G_{13}/E_2	G_{23}/E_2	ν_{12}	$SCF(k_4^2 = k_5^2)$
I	2.345	0.289	0.289	0.289	0.32	5/6
II	14.3	0.5	0.5	0.5	0.3	5/6

Table 7 Comparison of present DR, DR results of [4], finite element results [5] and experimental results [6] for a uniformly loaded simply supported (SS2) square orthotropic plate made of material I (*h/a* = 0.0115)

$\bar{q}$	$\bar{w}_c(1)$	$\bar{w}_c(2)$	$\bar{w}_c(3)$	$\bar{w}_c(4)$
17.9	0.5859	0.5858	0.58	0.58
53.6	1.2710	1.2710	1.30	1.34
71.5	1.4977	1.4977	1.56	1.59
89.3	1.6862	1.6862	1.74	1.74

S (1): present DR results (5×5uniform non – interlacing mesh over quarter of the plate).
S (2): DR results of [4].
S (3): Reddy's finite element results [5].
S (4): Zaghloul's and Kennedy's [6] experimental results as read from graph.

Table 8 Deflection of the center of a two – layer anti symmetric cross ply simply supported in – plane fixed (SS4) strip under uniform pressure (*b/a*= 5,*h/a*= 0.01)

$\bar{q}$	S	$\bar{w}_1[0^o/90^o]$	$\bar{w}_2[90^o/0^o]$	$\bar{w}_o(B_{ij} = 0)$	%(1)	%(2)	%(3)
18	1 2	0.6851 0.6824	0.2516 0.2544	0.2961	131.4 130.5	- 15.0 - 14.1	172.3 168.2

	3	0.6800	0.2600				
36	1	0.8587	0.3772		88.1	- 17.4	127.7
	2	0.8561	0.3822	0.4565	87.5	- 16.3	124.0
	3	0.8400	0.3900				
72	1	1.0453	0.5387		61.0	- 17.0	94.0
	2	1.0443	0.5472	0.6491	60.9	- 15.7	90.8
	3	1.0400	0.5500				
108	1	1.1671	0.6520		50.0	- 16.2	79.0
	2	1.1675	0.6630	0.7781	50.0	- 14.8	76.1
	3	1.1500	0.6600				

S (1): present DR results
S (2): DR results [8].
S (3): Values determined from sun and chin's results [7].
(1): $100 \times (\bar{w}_1 - \bar{w}_o)/\bar{w}_o$
(2): $100 \times (\bar{w}_2 - \bar{w}_o)/\bar{w}_o$
(3): $100 \times (\bar{w}_1 - \bar{w}_2)/\bar{w}_2$

Table 9 Center deflection of two – layer anti – symmetric cross – ply simply supported in – plane free (SS1) plate under uniform pressure and with different aspect ratios ($h/a = 0.01; \bar{q} = 18$).

b/a	S	$\bar{w}_1[0^o/90^o]$	$\bar{w}_2[90^o/0^o]$	$\bar{w}_o(B_{ij} = 0)$	%(1)	%(2)	%(3)
2.5	1	0.8325	0.8422	0.3907	113.1	115.6	- 1.15
	2	0.8328	0.8424	0.3907	113.2	115.6	- 1.1
2.0	1	0.7707	0.7796	0.3807	102.4	104.8	- 1.14
	2	0.7712	0.7799	0.3807	102.6	104.9	- 1.1
1.75	1	0.7173	0.7248	0.3640	97.0	99.1	- 1.0
	2	0.7169	0.7251	0.3640	97.0	99.2	- 1.1
1.5	1	0.6407	0.6460	0.3335	92.1	93.7	- 0.82
	2	0.6407	0.6455	0.3325	92.7	94.1	- 0.70
1.25	1	0.5324	0.5346	0.2781	91.4	92.2	- 0.4
	2	0.5325	0.5347	0.2782	91.4	92.2	- 0.4
1.0	1	0.3797	0.3797	0.1946	95.1	95.1	0.0
	2	0.3796	0.3796	0.1949	94.8	94.8	0.0

S (1): present DR results
S (2): DR results [8].
(1): $100 \times (\bar{w}_1 - \bar{w}_o)/\bar{w}_o$
(2): $100 \times (\bar{w}_2 - \bar{w}_o)/\bar{w}_o$
(3): $100 \times (\bar{w}_1 - \bar{w}_2)/\bar{w}_2$

V. CONCLUSIONS

A Dynamic relaxation (DR) program basedon finite differences has been developed for large deflection analysis of rectangular laminated plates using first order shear deformation theory (FSDT). The displacements are assumed linear through the thickness of the plate. A series of new results for uniformly loaded thin, moderately thick, and thick plates with simply supported edges have been presented. Finally a series of numerical comparisons have been undertaken to demonstrate the accuracy of the DR program. These comparisons show the following:-

1. The convergence of the DR solution depends on several factors including boundary conditions, meshes size, fictitious densities and applied load.
2. The DR large deflection program using uniform finite differences meshes can be employed with confidence in the analysis of moderately thick and flat isotropic, orthotropic or laminated plates under uniform loads.
3. The DR program can be used with the same confidence to generate small deflection results.
4. The time increment is a very important factor for speeding convergence and controlling numerical computations. When the increment is too small, the convergence becomes tediously slow; and when it is too large, the solution becomes unstable. The proper time increment in the present study is taken as 0.8 for all boundary conditions.
5. The optimum damping coefficient is that which produces critical motion. When the damping coefficients are large, the motion is over – damped and the convergence becomes very slow. At the other hand when the coefficients are small, the motion is under – damped and can cause numerical instability. Therefore, the damping coefficients must be selected carefully to eliminate under – damping and over – damping.

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