



بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

مقدمة

سيتم في هذا الجزء استكمال كتاب معادلات في

الرياضيات الهندسية والذي يحتوي على صيغ

وتعريفات لمعادلات هامة يمكن ان تفيد الباحثين في

مجال الرياضيات الهندسية

السفياني

*Derivatives*

$(\bar{c}) = 0, c = \text{constant.}$
$(\bar{x}) = 1$
$(\bar{x}^n) = n x^{(n-1)}$
$[\frac{1}{x}] = \frac{-1}{x^2}, [\frac{1}{x}]^{(n)} = (-1)^n \frac{n!}{(x^{(n+1)})}$
$(\sqrt{x}) = \frac{1}{(2\sqrt{x})}$
$(\sqrt{x}) = \frac{1}{(2\sqrt{x})}$
$(\sqrt[n]{x}) = \frac{1}{(n\sqrt{x^{(n-1)}})}$
$(\bar{e}^x) = e^x, (\bar{e}^u) = e^u \cdot \bar{u}$
$(\bar{a}^x) = a^x \ln a$
$(\bar{\ln} x) = \frac{1}{x}, (\ln x)^{(n)} = \frac{(-1)^n * (n-1)!}{x^n}$
$(\bar{\log}_a x) = \frac{1}{x} \log_a e = \frac{1}{(x \ln a)}$
$(\bar{\sin} x) = \cos x, (\sin ax)^{(n)} = a^n \sin[ax + n \frac{\pi}{2}]$
$(\bar{\cos} x) = -\sin x, (\cos ax)^{(n)} = a^n \cos[ax + n \frac{\pi}{2}]$
$(\bar{\tan} x) = \frac{1}{(\cos^2 x)} = \sec^2 x$
$(\bar{\cot} x) = \frac{-1}{(\sin^2 x)} = -\text{cosec}^2 x$

$(\sec^{-1} x) = \tan x \sec x$
$(\operatorname{cosec}^{-1} x) = -\cot x \operatorname{cosec} x$
$(\sin^{-1} x) = \frac{1}{\sqrt{(1-x^2)}}$
$(\cos^{-1} x) = \frac{-1}{\sqrt{(1-x^2)}}$
$(\tan^{-1} x) = \frac{1}{(1+x^2)}$
$(\cot^{-1} x) = \frac{-1}{(1+x^2)}$
$(\sec^{-1} x) = \frac{1}{(x\sqrt{(x^2-1)})}$
$(\operatorname{cosec}^{-1} x) = \frac{-1}{(x\sqrt{(x^2-1)})}$
$(\bar{sh} x) = \cosh x, (\operatorname{cosech} x) = -\operatorname{cosech} x \coth x$
$(\bar{cosh} x) = sh x, (\operatorname{sech} x) = -\operatorname{sech} x \tanh x$
$(\bar{tanh} x) = \frac{1}{(\cosh^2 x)} = \operatorname{sech}^2 x$
$(\bar{coth} x) = \frac{-1}{(\sinh^2 x)} = -\operatorname{cosech}^2 x$
$[f(x) \pm g(x)] = \bar{f}(x) \pm \bar{g}(x)$
$[f \frac{\bar{g}}{g}(x)] = g(x) \bar{f}(x) - f(x) \bar{g} \frac{(x)}{[g(x)]^2}, g(x) \neq 0$
$y^{(n)} = (uv)^{(n)} = u^{(n)} v + c_1 u^{(n-1)} v^{(1)} + c_2 u^{(n-2)} v^{(2)} + c_3 u^{(n-3)} v^{(3)} + \dots + c_r u^{(n-r)} v^{(r)} + \dots + u v^{(n)}$ نظرية ليبتنز لحاصل ضرب دالتين

*Series Expansion Of Trigonometric And Hyperbolic Functions*

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots,$
$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \dots$
$\tan x = x + \frac{x^3}{3} + 2\frac{x^5}{15} + 17\frac{x^7}{315} + 62\frac{x^9}{2835} + \dots$
$\cot x = \frac{1}{x} - \frac{x}{3} - \frac{x^3}{45} - 2\frac{x^5}{945} - \frac{x^7}{4725} - \dots$
$\sin^{-1} x = x + \frac{x^3}{6} + 3\frac{x^5}{40} + 5\frac{x^7}{112} + \dots$
$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$
$\cos^{-1} x = \frac{\pi}{2} - \sin^{-1} x$
$\cot^{-1} x = \frac{\pi}{2} - \tan^{-1} x$
$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \dots$
$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \dots$
$\sinh^{-1} x = x - \frac{x^3}{6} + 3\frac{x^5}{40} - 5\frac{x^7}{112} + \dots$
$\tanh^{-1} x = x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \dots$

$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$
$\mathit{coth}^{-1} x = \frac{1}{x} + \frac{1}{(3x^3)} + \frac{1}{(5x^5)} + \frac{1}{(7x^7)} + \dots$
$a^x = 1 + \frac{(x \ln a)}{1!} + \frac{(x \ln a)^2}{2!} + \frac{(x \ln a)^3}{3!} + \dots$
$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \dots$
$\mathit{tanh} x = x - \frac{1}{3}x^3 + \frac{2}{15}x^5 - \frac{17}{315}x^7 + \dots$
$\mathit{coth} x = \frac{1}{x} + \frac{1}{3}x - \frac{1}{45}x^3 + \frac{2}{945}x^5$
$\mathit{cosh}^{-1} x = \ln 2x - \frac{1}{2} \cdot \frac{1}{(2x^2)} - \frac{[(1)(3)]}{[(2)(4)]} \cdot \frac{1}{(4x^4)} - \frac{[(1)(3)(5)]}{[(2)(4)(6)]} \cdot \frac{1}{(6x^6)} + \dots$

*Main Formula In Hyperbolic Trigonometry*

$sh\ x = \frac{(e^x - e^{-x})}{2}$
$\mathbf{cosh}\ x = \frac{(e^x + e^{-x})}{2}$
$\mathbf{tanh}\ x = \frac{(e^x - e^{-x})}{(e^x + e^{-x})}$
$\mathbf{cosech}\ x = \frac{1}{(sh\ x)}$
$\mathbf{sech}\ x = \frac{1}{(\mathbf{cosh}\ x)}$
$\mathbf{coth}\ x = \frac{1}{(\mathbf{tanh}\ x)}$
$\mathbf{cosh}^2\ x - sh^2\ x = 1$
$1 - \mathbf{tanh}^2\ x = \mathbf{sech}^2\ x$
$1 - \mathbf{coth}^2\ x = -\mathbf{cosech}^2\ x$
$sh(-x) = -sh\ x$
$\mathbf{cosh}(-x) = \mathbf{cosh}\ x$
$\mathbf{tanh}(-x) = -\mathbf{tanh}\ x$
$sh(x \pm y) = sh\ x\ \mathbf{cosh}\ y \pm \mathbf{cosh}\ x\ sh\ y$
$\mathbf{cosh}(x \pm y) = \mathbf{cosh}\ x\ \mathbf{cosh}\ y \pm sh\ x\ sh\ y$
$\mathbf{tanh}(x \pm y) = \frac{(\mathbf{tanh}\ x \pm \mathbf{tanh}\ y)}{(1 \pm \mathbf{tanh}\ x\ \mathbf{tanh}\ y)}$
$sh\ 2x = 2\ sh\ x\ \mathbf{cosh}\ x$

*Main Formula In Trigonometry*

$\sin^2 x + \cos^2 x = 1$
$\tan x = \frac{(\sin x)}{(\cos x)}$
$\cot x = \frac{1}{(\tan x)} = \frac{(\cos x)}{(\sin x)}$
$1 + \tan^2 x = \sec^2 x = \frac{1}{(\cos^2 x)}$
$\sin x = \frac{\sqrt{(1 - \cos^2 x)}}{(\sqrt{1 + \tan^2 x})} = \frac{(\tan x)}{(\sqrt{1 + \cot^2 x})} = \frac{1}{(\sqrt{1 + \cot^2 x})}$
$\cos x = \frac{\sqrt{(1 - \sin^2 x)}}{((\sqrt{1 + \tan^2 x}))} = \frac{1}{(\sqrt{(1 + \cot^2 x}))} = \frac{(\cot x)}{(\sqrt{(1 + \cot^2 x}))}$
$\sin(-x) = -\sin x$
$\cos(-x) = \cos x$
$\tan(-x) = -\tan x$
$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$
$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$
$\tan(x \pm y) = \frac{[\tan x \pm \tan y]}{[1 \mp \tan x \tan y]}$
$\cot(x \pm y) = \frac{[\cot x \cot y \mp 1]}{[\cot x \pm \cot y]}$
$\sin x + \sin y = 2 \sin \frac{(x+y)}{2} \cos \frac{(x-y)}{2}$
$\log_a(xy) = \log_a x + \log_a y, \log(x/y) = \log_a x - \log_a y$ $\log_c^a = \log_b^a \times \log_c^b$



$$\log_a x^n = n \log_a x; \log_a \sqrt[n]{x} = \frac{1}{n} \log_a x$$

$$\log_{10} x = 0.4343 \log_e x; \log_e x = 2.303 \log_{10} x$$

*Laplace Transform Table*

$F(S)$	$F(T), T \geq 0$
1	$\delta(t)$ unit impulse at $t=0$
$\frac{1}{s}$	$u_s(t)$ unit step at $t=0$
$\frac{1}{s^2}$	$t u_s(t)$ ramp function
$\frac{1}{s^n}$	$\frac{1}{(n-1)!} t^{(n-1)}, n$ is +ve
$\frac{1}{s} e^{-as}$	$u_s(t-a)$ unit step starting at $t=a$
$\frac{1}{s}(1-e^{-as})$	$u_s(t) - u_s(t-a)$ rectangular pulse
$\frac{1}{(s+a)}$	$e^{-at}$ exponential decay
$\frac{1}{(s+a)^n}$	$\frac{1}{(n-1)!} t^{(n-1)} e^{-at}, n$ is +ve
$\frac{1}{(s(s+a))}$	$\frac{1}{a}(1-e^{-at})$
$\frac{1}{[s(s+a)(s+b)]}$	$\frac{1}{ab} [1 - \frac{b}{(b-a)} e^{-at} + \frac{a}{(b-a)} e^{-bt}]$
$\frac{(s+\alpha)}{[s(s+a)(s+b)]}$	$\frac{1}{(ab)} [\alpha - \frac{[b(\alpha-a)]}{(b-a)} e^{-at} + \frac{[a(\alpha-b)]}{(b-a)} e^{-bt}]$
$\frac{1}{[(s+a)(s+b)]}$	$\frac{1}{(b-a)} [e^{-at} - e^{-bt}]$
$\frac{s}{[(s+a)(s+b)]}$	$\frac{1}{(a-b)} [a e^{-at} - b e^{-bt}]$

$\frac{(s+\alpha)}{[(s+a)(s+b)]}$	$\frac{1}{(b-a)}[(\alpha-a)e^{-at} - (\alpha-b)e^{-bt}]$
$\frac{1}{[(s+a)(s+b)(s+c)]}$	$\frac{e^{-at}}{[(b-a)(c-a)]} + \frac{e^{-at}}{[(c-b)(a-b)]} + \frac{e^{-ct}}{[(a-c)(b-c)]}$
$\frac{(s+\alpha)}{[(s+a)(s+b)(s+c)]}$	$\frac{(\alpha-a)}{[(b-a)(c-a)]}e^{-at} + \frac{(\alpha-b)}{[(c-b)(a-b)]}e^{-bt} + \frac{(\alpha-c)}{[(a-c)(b-c)]}e^{-ct}$
$\frac{\omega}{(s^2 + \omega^2)}$	<b>sin</b> $\omega t$
$\frac{s}{(s^2 + \omega^2)}$	<b>cos</b> $\omega t$
$\frac{(s+\alpha)}{(s^2 + \omega^2)}$	$\frac{\sqrt{(\alpha^2 + \omega^2)}}{\omega} \mathbf{sin}(\omega t + \phi); \phi = \mathbf{tan}^{-1} \frac{\omega}{\alpha}$
$\frac{[s \mathbf{sin} \theta + \omega \mathbf{cos} \theta]}{(s^2 + \omega^2)}$	<b>sin</b> $(\omega t + \theta)$
$\frac{1}{[s(s^2 + \omega^2)]}$	$\frac{1}{\omega^2}(1 - \mathbf{cos} \omega t)$
$\frac{(s+\alpha)}{[s(s^2 + \omega^2)]}$	$\frac{\alpha}{\omega^2} - \frac{\sqrt{(\alpha^2 + \omega^2)}}{\omega^2} \mathbf{cos}(\omega t + \phi); \phi = \mathbf{tan}^{-1} \frac{\omega}{\alpha}$
$\frac{1}{[(s+a)(s^2 + \omega^2)]}$	$\frac{e^{-at}}{(a^2 + \omega^2)} + \frac{1}{[\omega \sqrt{(\alpha^2 + \omega^2)}]} \mathbf{sin}(\omega t - \phi); \phi = \mathbf{tan}^{-1} \frac{\omega}{\alpha}$
$\frac{1}{[(s+a)^2 + b^2]}$	$\frac{1}{b} e^{-at} \mathbf{sin} bt$
$\frac{1}{[s^2 + 2\xi\omega_n s + \omega_n^2]}$	$\frac{1}{[\omega_n \sqrt{(1-\xi^2)}]} e^{(-\xi\omega t)} \mathbf{sin} \omega_n \sqrt{(1-\xi^2)t}$
$\frac{(s+a)}{[(s+a)^2 + b^2]}$	$e^{-at} \mathbf{cos} bt$
$\frac{(s+\alpha)}{[(s+a)^2 + b^2]}$	$\frac{\sqrt{((\alpha-a)^2 + b^2)}}{b} e^{-at} \mathbf{sin}(bt + \Phi); \phi = \mathbf{tan}^{-1} \frac{b}{(\alpha-a)}$

$\frac{1}{[s[(s+a)^2+b^2]]}$	$\frac{1}{(a^2+b^2)} + \frac{1}{(b\sqrt{(a^2+b^2)})} e^{-at} \cdot \sin(bt-\phi), \phi = \tan^{-1} \frac{b}{-a}$
$\frac{1}{[s(s^2+2\xi\omega_n s+\omega_n^2)]}$	$\frac{1}{\omega_n^2} - \frac{1}{(\omega_n^2\sqrt{(1-\xi^2)})} e^{(-\xi\omega_n t)} \cdot \sin(\omega_n\sqrt{(1-\xi^2)t+\phi}); \phi = \cos^{-1}\xi$
$\frac{(s+\alpha)}{[s[(s+a)^2+b^2]]}$	$\frac{\alpha}{(a^2+b^2)} + \frac{1}{b} \sqrt{\frac{[(\alpha-a)^2+b^2]}{(a^2+b^2)}} \cdot e^{-at} \sin(bt+\phi); \phi = \tan^{-1} \frac{b}{(\alpha-a)} - \tan^{-1} \frac{b}{-a}$
$\frac{1}{[[s+c](s+a)^2+b^2]}$	$\frac{e^{-ct}}{[(c-a)^2+b^2]} + \frac{[e^{-at} \sin(bt-\phi)]}{[b\sqrt{((c-a)^2+b^2)}]}; \phi = \tan^{-1} \frac{b}{(c-a)}$
$\frac{1}{[s^2(s+a)]}$	$\frac{1}{a^2}(at-1+e^{-at})$
$\frac{1}{[s(s+a^2)]}$	$\frac{1}{a^2}(1-e^{-at}-ate^{-at})$
$\frac{1}{[s(s+c)[(s+a)^2+b^2]}$	$\frac{1}{[c(a^2+b^2)]} - \frac{e^{-ct}}{[c[(c-a)^2+b^2]]} + \frac{(e^{-at} \sin(bt-\phi))}{[b\sqrt{(a^2+b^2)}\sqrt{((c-a)^2+b^2)}]}$ $\phi = \tan^{-1} \frac{b}{-a} + \tan^{-1} \frac{b}{(c-a)}$
$\frac{(s+\alpha)}{[s(s+c)[(s+a)^2+b^2]}$	$\frac{\alpha}{[c(a^2+b^2)]} - \frac{[(c-\alpha)e^{-ct}]}{[c[(c-a)^2+b^2]]} + \frac{\sqrt{((\alpha-a)^2+b^2)}}{[b\sqrt{(a^2+b^2)}\sqrt{((c-a)^2+b^2)}]}$ $\cdot e^{-at} \sin(bt+\phi); \phi = \tan^{-1} \frac{b}{(\alpha-a)} - \tan^{-1} \frac{b}{-a} - \tan^{-1} \frac{b}{(c-a)}$

**Z – Transform Table**

$e(t)$	$E(Z)$
$\delta(t)$	1
$\delta(t - nT)$	$z^{-n}$
$U_s(t)$	$\frac{z}{(z-1)}$
$t$	$\frac{Tz}{(z-1)^2}$
$t^2$	$T^2 z \frac{(z+1)}{(z-1)^3}$
$t^{(n-1)}$	$\lim_{a \rightarrow 0} (-1)^{(n-1)} \frac{\partial^{(n-1)}}{(\partial a^{(n-1)})} \left[ \frac{z}{(z - e^{(-aT)})} \right]$
$e^{-at}$	$\frac{z}{(z - e^{-at})}$
$\frac{1}{(b-a)}(e^{-at} - e^{-bt})$	$\frac{1}{(b-a)} \left[ \frac{z}{(z - e^{-at})} - \frac{z}{(z - e^{-bt})} \right]$
$\frac{1}{a}(u_s(t) - e^{-at})$	$\frac{1}{a} \left[ \frac{((1 - e^{-aT})z)}{((z-1)(z - e^{-aT}))} \right]$
$\frac{1}{a} \left[ t - \frac{(1 - e^{-at})}{a} \right]$	$\frac{1}{a} \left[ \frac{Tz}{(z-1)^2} - \frac{((1 - e^{-aT})z)}{[a(z-1)(z - e^{-aT})]} \right]$

محمد بن محمد اللّٰه ونوفيه

*Email*

[memory\\_code\\_84@yahoo.com](mailto:memory_code_84@yahoo.com)

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